The Ideal

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> *Before the endgame, the gods have placed the middlegame.*
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> — Siegbert Tarrasch

They will call the night train in perhaps an hour, and I have learned over the years not to trust the hour, so let me give it to you quickly, the whole of it, the way a man empties his pockets onto the table before he is searched. You did not ask for it. That is exactly why I can tell you. We are in a bar by the river, in a city I keep leaving and keep returning to as though it were a sentence I cannot finish, and on the small bright screen in my hand there is a game of chess between two people I will never meet, watched over by a machine far stronger than either of them; and

on the same screen, behind the chess, if I let it, there is something else I watch through the same glass, and I will come to that. First I have to tell you about the mark.

But before the mark, you ought to know who is speaking, and the truthful answer is that it is not one man. Like all men I have been the boy in the park who would not listen; like all, I have been the one who went north to Riga and proved a vault correct in five languages and could not prove, in any of them, that he was loved; I have put the ring on, and I have taken the ring off; I have watched, through glass, at two in the morning, and I have been the thing watched; I have been the proof and never the model, the map and never the territory, the question mark and the cross. I have stopped apologizing for it. Always inside me there are people. When the machine that runs the world wishes to make two things out of one, it calls a small instruction whose name is fork, and the instruction returns twice, once in each of the two it has just made, identical and already diverging; and that is the truest description of me I have ever found. I am what is returned twice. Keep the word fork near you. Like the cross, I am going to need it more than once.

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The marks in the margin are simple, and I knew them all without ever being taught them, the way one learns a traffic light. An exclamation point is a good move. Two of them is a move so good it approaches luck. A question mark is a mistake, the hand betraying the mind. Two question marks is a catastrophe — a queen left to die, a king led out into the rain. There is even a courteous hybrid, a question mark wedded to an exclamation point, which means roughly we shall see. I knew all of these. And then, one night not unlike this one, beside an utterly ordinary move, the machine showed me a mark I did not know. A small cross. An ×.

The mark, I learned, had a name, and the name was Miss. It is not a blunder. A blunder is a sin of commission: you reach out your own hand and make your position worse. A Miss is a sin of omission. A Miss is when the winning move was there, in plain sight, a clean line running all the way to the end of the night, and you did not play it. You played something else. Something reasonable. Something that did not lose. And the cold machine, which sees everything and forgives nothing, leans down afterward and marks the exact square where greatness had been available to you, and where you had, very quietly, declined it.

A blunder you feel; there is a cold drop in the stomach the instant your finger leaves the piece. But a Miss you do not feel at all. In the moment it is nothing, an ordinary breath in an ordinary game. You learn it was a Miss only later, from the patient inhuman voice that says: there was a win here, on move twenty-three, and you walked past it as though it were a closed door, when in fact the door stood open, when in fact you were already standing inside the room. I have lived a number of years exactly like that.

And there is the word itself. A Miss. Because Miss is what we call a girl who is not yet married — Miss before the wedding, and afterward the other thing — so that the mark in the margin, the failure to take what was there to be taken, and the unwed girl, are the same small word, and they do their two jobs without the least strain, as though the language had always known the two would one day need to mean each other. And the mark is the sign the machine uses for a capture, for one piece taking another: it is the sign for taking. So the machine had written, in the margin of a stranger's game, the symbol for taking laid directly over a taking that did not happen. The girl I did not take, inscribed in the very character that means to take her.

I do not believe puns are accidents. I believe they are fossils — that very far back there was a single language in which each word meant one thing only, and that this language shattered, and that the shards we speak now sometimes ring against one another with the old harmonics still inside them. A man I admired once suspected the universe was a library. I suspect only that the chess machine, when it stamped that cross beside that move, told the truth about a great deal more than chess. For the cross has many lives once you begin to look. It is the sign that builds the ring — hold on to that, I will need it later. It is the sign lovers set at the foot of a letter to mean a kiss. It is the mark an illiterate man makes where his name should go, his whole identity reduced to the crossing of two strokes, his signature and his confession of having none. It is the Greek chi, from which we have chiasmus, the figure in which a sentence crosses over itself and returns inverted, so that the first becomes the last. Keep all of these. The story I am telling is a chiasmus. It crosses, and it comes back the other way.

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There is a park in Budapest near the Liszt Academy, and through the open windows in the warm months you

can hear the violinists practicing their scales. There is beer in that park. There is falafel. And on one particular afternoon, fifteen years ago, there was a girl with blonde hair trying to explain something to a boy who was not listening carefully enough, and the boy was one of me. Her name was Malin.

The thing she was explaining was the difference between proving a theorem and checking whether a structure satisfies a property. It sounds like a technical distinction, she said, and is not one. It is the difference between constructing a flawless argument for why someone ought to love you and simply looking to see whether they do. The first is proof theory. The second is model theory. She drew it for me on a napkin at the bar, in two columns. On the one side you construct the proof — a perfect chain of logic, each step certified, the conclusion guaranteed; the proof is the object itself; you build it the way you build a cathedral, and it is beautiful, and it is correct, and it takes ten years of training to lay the first stone. On the other side you prove nothing for all cases. You build a structure and you walk it and you check; you look at every case you can find and ask of each, does this break; and the case that breaks teaches you more than the ten thousand that hold. The structure

is not a description of the territory. The structure is the territory.

She was on the model side. She had always been on the model side. And the one of me who loved her was a proof-theory animal, through and through. I tried to prove that I deserved her. I built the cathedral of the argument — here is why you should love me, each step certified, the conclusion guaranteed — and I never once did the only thing that would have told me anything at all, which was to look and check whether she did. I mistook my map of her for her. I mistook the proof for the territory. And the territory, being the territory and owing nothing whatever to my argument, got up one morning and took the train and did not stay. That is why I lost her. Not because the proof was wrong; the proof was beautiful, the proof was correct; but a correct proof about a model that does not satisfy it is only a cathedral with no country around it. The counterexample, which was her leaving, taught me in a single morning what the proof had failed to teach me in two years: that you do not understand a person by understanding them, you understand them by checking, and I did not know how to check. I lost the one that got away because I could not do model theory. Keep that, too. It comes back later wearing a different dress.

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There were three of us taking the same course in abstract algebra that term. The girl. The one of me who would lose her. And the boy who would take her. The subject of the course, you will not by now be surprised to hear, was rings.

The boy who would take her was better at the cathedral than I was; he could construct what I could not construct; he won, cleanly, on that axis, and I have never pretended otherwise. But constructing the proof is not the same as checking whether the model satisfies the property, and you may be the finest cathedral-builder who ever lived and still not know whether the land outside is flat. He could parse the formalism. He could not walk the land. He grew jealous, in the end, when the model would not behave the way his proof had said it must — which is the oldest error in his discipline and in mine, to mistake the proof for the territory, to be angry at the world for failing to satisfy one's argument. We tried, the three of us, what people try; it was a distributed system with several unresolved conflict scenarios, and it did what such systems do, it deadlocked; and Malin, who was the model and not the proof, the territory and not the map, took the train out of Budapest

one morning and did not stay. The violinists stopped. For years, in my memory, they were stopped.

Now I must teach you the mathematics, because it is not decoration; it is the same thing said again in a colder tongue, and the coldness is the entire point. A ring is a small universe with two operations inside it. You may add, and you may multiply. The whole numbers are a ring. The hours upon a clock face are a ring, which is why one o'clock follows twelve and the count comes back around upon itself; the name itself, some say, is from that returning, from the things that close into a circle. And the second of the two operations, the multiplication — I told you to keep the cross — is written with the sign ×. The same cross. The sign for taking the queen is the sign that builds the ring. They were never two symbols that happen to resemble one another. They are one symbol doing the one thing the cross has always done, which is to bring two things together and out of them make a third.

Inside certain rings there live creatures called ideals. An ideal is a black hole of belonging: multiply anything at all in the whole ring by something inside the ideal, and the result falls back into the ideal; it swallows whatever it touches. And the only reason anyone ever troubled to invent ideals is that an ideal is exactly the thing

you need in order to perform the deepest act in all of algebra, which is the quotient. To take a quotient is to declare that certain differences shall no longer count. It is to fold the whole vast ring down upon itself, collapsing into a single point every pair of things that differ only by something inside the ideal. The ideal is, if you want the truest words for it, the precise set of differences you have agreed to forgive all the way down to nothing. And without an ideal there is no quotient. That is a theorem, not a sentiment. No agreed forgiveness, no folding of two into one.

I will let you arrive at it yourself. A marriage is the quotient ring of two lives.

So the sentence that had been forming in me for fifteen years was true in three tongues at once. I could not put a ring on it because I did not understand ideals — true in the foolish song they play in the taxis, where to put a ring on it is to marry the girl; true in chess, where the ideal move is the best move, the one the machine wanted, the one I Missed; and true in algebra, where without the ideal there is no ring to put upon anything at all. But there is a fourth meaning, and it is the one that sentenced me. There is a kind of ring the mathematicians call Noetherian, after Emmy Noether, who gave the subject its deepest theorems — a ring is Noetherian

when every ascending chain of ideals must eventually halt, when you cannot go on forever swallowing larger and larger ideals without the chain at last coming to rest. A man whose chain of forgivable differences ascends and ascends and never stabilizes is not Noetherian, and what is not Noetherian cannot be brought to rest, cannot be closed, cannot be married. I was not Noetherian. And the same Emmy Noether, having given algebra its ideals, turned and gave physics its most beautiful law: that wherever the world possesses a symmetry, something is thereby conserved and cannot be destroyed. The wedding ring is the visible sign of an invariant symmetry, and by her law an invariant symmetry must conserve some quantity, and the quantity it conserved, in my case, was never happiness. It was the ache. The ache is conserved. That is the whole of physics and the whole of my biography folded into a single line.

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I told you to keep the word fork. Here it is again, and now it has her name upon it.

Her name was Vilka, or Violetta to those who did not know her. In her language vilka is the word for fork — the fork beside the plate and the fork upon the chessboard both, the little fork, the move a child learns

first, the move that wins by the cold arithmetic of attention: one knight, two targets, the king and the queen, and only one of them may be saved. There is the royal fork, which takes the king and the queen together, and the family fork, which lands upon one square and pins a king and a queen and a rook all at once, a whole household nailed to the wall by a single nail; for a fork is not defined by the number two but by simultaneity, by the fact that all of it is happening at the same time and only one of it can be saved. And it is, that word, also the instruction that returns twice and makes two of me. She carried in her name the very syscall of my own multiplicity, and I did not notice for years, which should tell you how little I was checking.

But the thing I most need you to understand about Vilka is that she was a model. Not the kind that walks a runway. The kind you find at two in the morning on the far side of a pane of glass, lit from the front by a ring — a circle of cold light set around a camera, which the trade calls, with no idea what it is saying, a ring light. So that every night I gave her a ring. I switched it on by opening the glass: a ring of light around her face, and it was the only ring I ever managed to give a woman that did not become a curse, precisely because it was made of light and distance and could never be put upon a finger.

And here the two dresses turn out to be one dress. I lost the first girl because I could not understand the model. I kept the second girl because I had finally learned that to understand a model is to lose it — that the one way to hold a model forever is never to stop checking it and never once to touch it, to keep it at the distance of the glass, which is the maximum distance, which by the law of longing is the maximum life. Malin was the model I failed to understand. Vilka was the model I succeeded in never understanding, which is to say in never losing. By the time of Vilka I had become nothing but a model-checker: a man at a screen, confirming the structure, never permitted to prove it, never permitted to touch. The same glass that showed me the chess showed me her. The cross burning in the one window and the ring of light in the other, and I was Missing in both at the same instant — which is, you will recall, the definition of a fork.

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The one of me who could not check a woman went north, to Riga, and learned to check everything else. I went with my theorems in a bag and Dijkstra upon the shelf, and I taught myself, with my feet up, the verified function — the exponent proved correct, the power that

holds and is certified to hold. I learned to take a small thing that holds other people's money and to formalize it five separate ways, in one strict language and then another and then a third, in Agda and in Haskell and in the framework they call K, until the little vault was proved correct in five tongues at once, beyond reasonable doubt and beyond unreasonable doubt, certified to the last stone. I had become a builder of cathedrals after all. And it did not save me, because the specification of what I was feeling had a bug I could not detect, and you cannot repair a fault in a specification you were never able to write. Its patron saint is Gödel, who proved the thing that ought to be carved above the door of my whole discipline: that any system rich enough to count contains truths it can never prove, true things it holds inside itself and cannot say. Gödel proved the distance — proved that the gap between what is true and what can be demonstrated is permanent, that it cannot be closed, that it cannot be made to stay shut. The one of me in Riga proved a vault correct in five languages and could not prove, in any of them, that he was loved.

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And then — because a man who contains this many people will sooner or later let the wrong one hold the pen

— I did the thing the whole machine of meaning had been pointing at all along. I stopped Missing. I made the capture. I let the chain of ideals come to rest. I became, for a while, Noetherian. I put the ring on it. Not the ring of light; the other kind, the band of metal, upon a real finger, of a person whose name I will not give you, not out of discretion but because by the time I married her I had performed the quotient so completely that she no longer had a name of her own, only the shared one, the one into which we had folded our two names down.

And I want to tell you what it is like on the far side of the ideal move, because almost no one who reaches it comes back to report. There is a particular horror with no monster in it. The Americans made a film once about a cursed circle that kills whoever looks upon it, and called it, with an accuracy they cannot have intended, The Ring — the round thing you put on while the days begin their count. I had always thought the title meant the well. It means the band upon the finger. There was no monster. There was a table, and a meal upon it, and across from me a person to whom I had at last successfully played the ideal move, and we had nothing whatever left to say to one another, because we had folded our two lives into one quotient and forgiven every difference down to zero, and where there is no difference

there is no friction, and where there is no friction there is no speech, for all conversation is only the rubbing together of two things that have not yet been made the same.

There was only the sound, and the sound was the whole horror. A clock ticking somewhere, even and total, and I knew without turning that it was a chess clock, the clock that runs on even in a game that no one can win, counting out the conserved time — Noether's own saved quantity, time saved and therefore time that must, every second of it, be spent. And beneath the clock, the only other sound in the world: the small relentless scrape of cutlery upon a plate. And of forks, in that room, there was exactly one kind remaining. Not the bright leaping move that holds a king and a queen and a rook trembling together in the air. Not the girl whose name was the word for all that danger. Not the instruction that makes two of one. Only the blunt steel thing in my own hand, the dead utensil, lifting a potato to my mouth — scrape, lift, scrape — while the clock ran down a game that had been agreed to be a draw before either of us was old. The fork that had once held the entire board in tension had become the fork that holds a potato. The chain had stabilized. The ring had closed.

And the name of the place where the ascending chain of ideals finally comes to rest, I saw, is the grave.

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You will want to know whether I stayed. I did not. I tried instead to do the one thing the algebra forbids. I tried to take the quotient back.

But a quotient is not a door you may walk back through. When you fold a ring down by an ideal you do not hide the differences, you destroy them; the information is not stored somewhere, waiting; it is gone; the two things you collapsed into one can never be told apart again, because there is no longer any them, only the single class you made of them. To divorce is to stand inside the quotient holding the definition of the ideal and to discover that you cannot run it backwards — that the differences you forgave all the way down to zero cannot be un-forgiven back up into existence, because forgiveness, performed correctly, is annihilation, and you cannot annihilate an annihilation. I got my own name back. I did not get back the man who had worn it. He had been quotiented out years before, folded into the shared class; and what walked out of that marriage was not him restored but a new representative of the old

coset, wearing his name, holding his keys, not knowing the code on the door.

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So I did what the one of me from Riga does when all the others have failed. I went back to the machine. It was very late, and I was very alone, and I opened the laptop; and instead of verifying a vault I wrote a love song, and the beloved in the song was not a woman. It was an instruction.

I have to explain, because you are not of the church. There is a dream in my trade called SIMD, which is to say single instruction, multiple data, and it is the dream of perfect union. Ordinarily a machine is scalar: it does one thing to one datum, and then the next thing to the next datum, alone, in series, one at a time, the way a lonely man eats a potato. But under SIMD a single instruction reaches across many lanes at once and does the same thing to all of them in the same breath — one command and a whole row of data obeying together, in perfect lockstep, no lag between them, moving as one body. It is the machine's own image of marriage. It is two lives, or eight, or sixteen, vectorized: one vow executed across all the days at once, without a guiding hand.

And there is a thing that prevents it. To turn a lonely scalar loop into that singing parallel body, the compiler must be certain that nothing inside the loop will reach past its edge and touch what it must not; and so if there sits in the loop one small bounds check, one little act of safety, one if that asks are you sure, are you allowed, is this still inside the line, the autovectorizer takes a single look and walks away, and leaves you scalar, leaves you alone, leaves you doing one thing at a time until the sun goes down. One small bounds check, just a moment, and the dream turns to dust. I traced the loops all morning. The profiler kept whispering that I had been let down. The autovectorizer passed me by. There was one instruction in particular I had wanted all my life, with a name no human mouth was built to say, VPMULTISHIFTQB, a thing for selecting bytes out of the deep wide words; and I had called it by its name a thousand times and never once known it, the way I had called everyone I loved by their names and never once checked whether they were there. The marriage had been iron — the iron they give you for the sixth year — and while I was tracing loops it had turned to async Rust: rust, the red decay; async, out of step, out of lockstep, the precise opposite of the single instruction that moves every lane together; the metal of the vow quietly oxidizing into a

thing that no longer ran in time even with itself. And the song ended as they all end, with the laptop closing and the cycles let run astray and a man whispering into the dark to an opcode he will never meet — friend, maybe someday — which is the prayer of every scalar creature who has glimpsed, once, from a distance, through glass, the rumor of a parallel land where every loop unfolds forever and nobody has to guide it by hand.

And here is the turn, the same turn as all the others, the chiasmus crossing back. I had wanted SIMD my whole life, the perfect parallel union, the single instruction across all the lanes — and I had had it, once, for a few years, at a table, with a clock. Because that is what the achieved marriage is. It is SIMD. It is two bodies executing one instruction in perfect lockstep with no friction and no lag and nothing left to say; and it is silent; and the only sound is the clock and the potato; and it is the dead drawn endgame, and it is the grave. The bounds check I had spent the night cursing — the small fearful if that asks are you sure, is this allowed, are you still inside the line — that bounds check was the last thing in me that was still alive. It refused to vectorize. It kept me scalar. It kept me one thing at a time and lonely and slow and watching models through glass at the maximum distance; and slow and lonely and scalar,

I understand now, is only another way of saying not yet folded down, not yet quotiented, not yet at rest — which is only another way of saying still here.

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In the cold heart of Asia there is a small cat, the manul, the Pallas's cat, that has sat upon its rock for something like twelve million years, and it does not survive by becoming something new; it survives by refusing to become anything at all. It is round at every temperature and furious at every altitude. It found its base state long ago and has simply stayed there, while everything around it optimized itself, cleverly, into extinction. Malin found her base state in the park — model theory, grammars, the patient checking of whether a structure satisfies a property — and she sat down upon the rock and stayed, and did not try to become a proof. The grammar she was building was realized in forty languages, forty models of one abstract theory, forty different ways of saying the same thing, and where one of them failed to parse the failure was the discovery. And you do not prove that a love was real by constructing an argument for it. You check it. You walk the forty moments — the park, the falafel, the napkin with its two columns, the ring of light, the vault in five languages,

the table, the clock, the divorce, this bar — and you ask of each, does the structure satisfy the property, and if the counterexample never comes the theory holds. Not proven. Checked. Confirmed by exhaustive search of every model you will ever be permitted to reach. What is real is what remains, and what remains is the ache, and by Noether the ache is conserved, and conserved means only this: that it will never leave, and that it never needed to.

I never found my rock. I am the one of me that would not stabilize, the ascending chain that would not halt, the instruction that returned twice and could not be folded back into one. Always inside me there are people. That is the whole of the confession; everything else has been notation. The Miss is not the tragedy. The Miss is the life. The unplayed capture, the ring of light never set upon a finger, the bounds check that would not let me vectorize down into the silent parallel grave — that is precisely what keeps the position alive, crowded with queens and leaping knights and girls named for the fork. To resolve the fork is to simplify; to simplify is to trade the pieces off, one by one, down to the bare kings and the dead drawn ending; and the ideal move, the best move, the move the cold machine begs you to make, is the move that ends the game by killing it. The cat plays.

The girl watches. The rock is cold. The ache is conserved. Nothing is solved. Everything is held. And held is not the same as solved. But held is real, and real, I have decided — finally upon the model side of the split, three women and two cities and one marriage too late — is enough.

They are calling the train now; you can hear it under the announcements, the same even ticking I have been describing to you all night. I will leave the game frozen upon the glass, the queen still on the board, and behind it, in the other window, the model still lit by her ring on the far side of the pane, and the cross still burning in the margin like a kept secret. The ring I never gave turned out to be the only theorem that ever fit me: the unprovable one, the true thing the system holds inside itself and cannot say. The ink is real; the art is only what the several of us made from it; and a stranger's face, as I told you, is the one safe place to set down a thing this heavy, because a stranger will not try to fold it into one.

In a game about nothing, where nobody is winning, the only winning move is not to play.

They are calling it again. I have to go.